1 The position vectors of three points A, B and C relative to an origin O are given by

$$\mathbf{a} = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k},$$
$$\mathbf{b} = 2\mathbf{i} + 3\mathbf{j},$$

and
$$\mathbf{c} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

respectively.

(a) Find:

(i) $\mathbf{a} \times \mathbf{b}$; *(2 marks)*

(ii) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$. *(2 marks)*

(b) State a geometrical relationship between the points O, A, B and C. *(1 mark)*

3 Given that

$$\mathbf{b} \times \mathbf{c} = \mathbf{i} \quad \text{and} \quad \mathbf{c} \times \mathbf{a} = 2\mathbf{j},$$

express

$$(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} + \mathbf{b} + 5\mathbf{c})$$

in terms of $\mathbf{i}$ and $\mathbf{j}$. *(6 marks)*

Q	Solution	Marks	Total	Comments
1 (a)(i)	$\mathbf{a}\times\mathbf{b} = \begin{bmatrix}3\\4\\-2\end{bmatrix}\times\begin{bmatrix}2\\3\\0\end{bmatrix}=\begin{bmatrix}6\\-4\\1\end{bmatrix}$	M1A1	2	
(ii)	$(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}=\begin{bmatrix}6\\-4\\1\end{bmatrix}\bullet\begin{bmatrix}1\\2\\2\end{bmatrix}=6-8+2=0$	M1A1F	2	
(b)	O, A, B and C and are coplanar	E1	1	no ft here
	Total		**5**	

3	Sensible expansion	M1		if $\mathbf{a}^2 + \mathbf{ab} + 5\mathbf{ac}$... used M0 unless some indication of understanding, e.g. $\mathbf{a}^2 = 0$.
	Cancelling out $\mathbf{a}\times\mathbf{a}$ etc.	M1		$\mathbf{b}=\mathbf{j}$, $\mathbf{a}=2\mathbf{i}$, $\mathbf{c}=\mathbf{k}$ or similar: 0
	Cancelling out $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{a}$	M1		
	$\mathbf{a}\times 5\mathbf{c}+\mathbf{b}\times 5\mathbf{c}$	A1		} All A's depend on M3
	$5\mathbf{i}-10\mathbf{j}$	A1A1	6	
	Total		**6**	

5 The position vectors **a**, **b** and **c** of three points A, B and C are

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix},$$

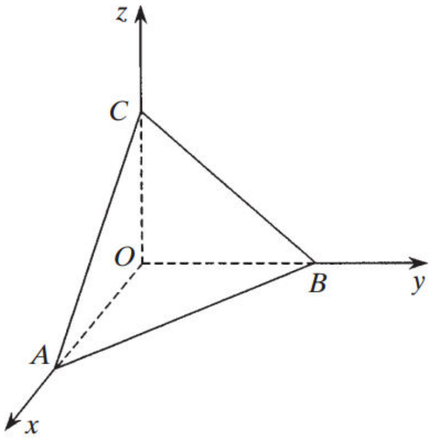
respectively.

(a) Calculate $(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})$. *(4 marks)*

(b) Hence find the exact value of the area of the triangle ABC. *(3 marks)*

5 (a)	$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$ seen $\mathbf{c}-\mathbf{a}$ $\quad$ $\mathbf{b}-\mathbf{a}$	B1		Either
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & -1 \\ 1 & -2 & 1 \end{vmatrix}$	M1		
	$\mathbf{i}+\mathbf{j}+\mathbf{k}$ or $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	A1A1	4	A1 1 correct (or all –ve) ft A1 all correct
(b)	$\frac{1}{2}\lvert(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})\rvert$	M1		$\frac{1}{2}$ prev. result
	$\frac{1}{2}\sqrt{1^2+1^2+1^2} = \frac{1}{2}\sqrt{3}$	M1A1	3	$\sqrt{}$ M1 A1 CAO allowing –ves in (a)
	Total		**7**	

6 The four vertices of a tetrahedron $OABC$ are at the points with position vectors $\mathbf{0}$, $a\mathbf{i}$, $b\mathbf{j}$ and $c\mathbf{k}$.



(a) Express $(b\mathbf{j} - a\mathbf{i}) \times (c\mathbf{k} - a\mathbf{i})$ in the form $p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$. *(3 marks)*

(b) Hence show that the area of the triangular face ABC is

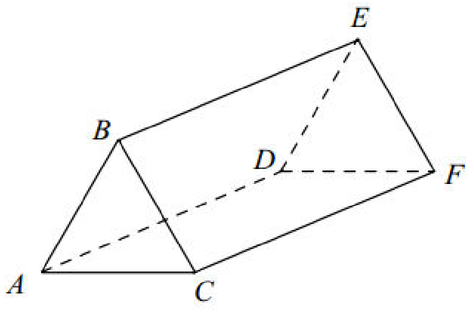
$$\left[\left(\frac{ab}{2}\right)^2 + \left(\frac{bc}{2}\right)^2 + \left(\frac{ac}{2}\right)^2\right]^{\frac{1}{2}}.$$

(4 marks)

6 **(a)**	$b\mathbf{j}\times c\mathbf{k} - a\mathbf{i}\times c\mathbf{k} - b\mathbf{j}\times a\mathbf{i} + a\mathbf{i}\times a\mathbf{i}$ $= bc\mathbf{i} + ac\mathbf{j} + ab\mathbf{k}$	M1 M1A1	3	a.e.f
(b)	Area $= \frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|$	B1√		
	$= \frac{1}{2}(b\mathbf{j} - a\mathbf{i})\times(c\mathbf{k} - a\mathbf{i})$			[√ on –ve signs from (a)]
	$= \left\|\frac{bc}{2}\mathbf{i} + \frac{ac}{2}\mathbf{j} + \frac{ab}{2}\mathbf{k}\right\|$	M1A1√		
	$\left[\left(\frac{ab}{2}\right)^2 + \left(\frac{bc}{2}\right)^2 + \left(\frac{ac}{2}\right)^2\right]^{\frac{1}{2}}$	A1√	4	a.g.
(c)	$(\text{Area } \Delta ABC)^2 =$ $(\Delta OAB)^2 + (\Delta OAC)^2 + (\Delta OBC)^2$	E2	2	E1 if recognises $(\text{area } \Delta ABC) = \frac{ab}{2}$
	Total		**9**	

6 The triangular prism *ABCDEF* has parallel triangular ends *ABC* and *DEF*, and the edges *AD*, *BE* and *CF* are parallel.

The coordinates of *A*, *B*, *C* and *D* are $(1, 2, 0)$, $(-1, 2, p)$, $(3, 0, 2)$ and $(4, 1, 5)$ respectively.



(a) Find the coordinates of the point *E* in terms of *p*. *(2 marks)*

(b) (i) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. *(3 marks)*

(ii) Hence find $\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})$, giving your answer in terms of *p*. *(2 marks)*

(c) (i) Determine the volume of the prism *ABCDEF* when $p = 1$. *(2 marks)*

(ii) Describe the configuration of *ABCDEF* when $p = -4$. *(2 marks)*

Q	Solution	Marks	Total	Comments
6 (a)	$\overrightarrow{AD} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$	B1		at any stage
	$\therefore E$ is $(2, 1, p+5)$	B1F	2	
(b)(i)	$\overrightarrow{AB} = \begin{bmatrix} -2 \\ 0 \\ p \end{bmatrix}$ $\quad \overrightarrow{AC} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$	B1		
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} -2 \\ 0 \\ p \end{bmatrix} \times \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2p \\ 4+2p \\ 4 \end{bmatrix}$	M1A1F	3	
(ii)	$\overrightarrow{AD} \cdot \overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} 2p \\ 2p+4 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$			
	$= 6p - 4 - 2p + 20$	M1		
	$= 4p + 16$	A1F	2	
(c)(i)	$p = 1 \quad \overrightarrow{AD} \cdot \overrightarrow{AB} \times \overrightarrow{AC} = 20$	B1F		
	Volume of prism is 10	B1F	2	
(ii)	$p = -4 \quad \overrightarrow{AD} \cdot \overrightarrow{AB} \times \overrightarrow{AC} = 0$	B1		must equal zero for this B1
	$\therefore \ \overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar	E1	2	OE
	Total		**11**	