FP4 - Vector Product Challenge

Challenge 1

The position vectors of three points A, B and C relative to an origin O are given by

$$\mathbf{a} = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k},$$
$$\mathbf{b} = 2\mathbf{i} + 3\mathbf{j},$$

and

$$\mathbf{c} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

respectively.

(a) Find:

(i) $\mathbf{a} \times \mathbf{b}$; *(2 marks)*

(ii) $(\mathbf{a} \times \mathbf{b}) . \mathbf{c}$. *(2 marks)*

(b) State a geometrical relationship between the points O, A, B and C. *(1 mark)*

Given that

$$\mathbf{b} \times \mathbf{c} = \mathbf{i} \quad \text{and} \quad \mathbf{c} \times \mathbf{a} = 2\mathbf{j},$$

express

$$(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} + \mathbf{b} + 5\mathbf{c})$$

in terms of $\mathbf{i}$ and $\mathbf{j}$. *(6 marks)*

Challenge 2

The position vectors **a**, **b** and **c** of three points A, B and C are

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix},$$

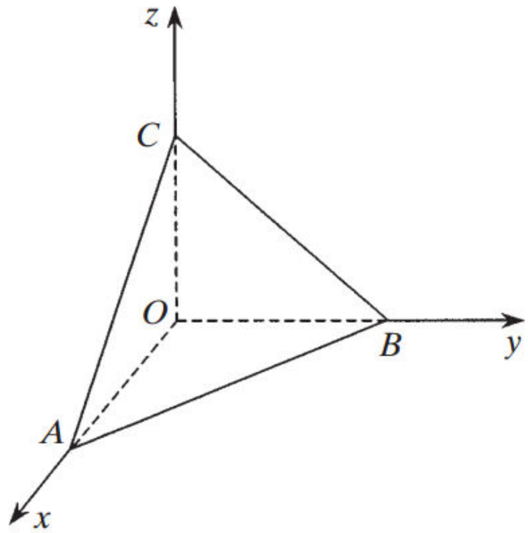
respectively.

(a) Calculate $(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})$. *(4 marks)*

(b) Hence find the exact value of the area of the triangle ABC. *(3 marks)*

Challenge 3

The four vertices of a tetrahedron $OABC$ are at the points with position vectors $\mathbf{0}$, $a\mathbf{i}$, $b\mathbf{j}$ and $c\mathbf{k}$.



(a) Express $(b\mathbf{j} - a\mathbf{i}) \times (c\mathbf{k} - a\mathbf{i})$ in the form $p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$. *(3 marks)*

(b) Hence show that the area of the triangular face ABC is

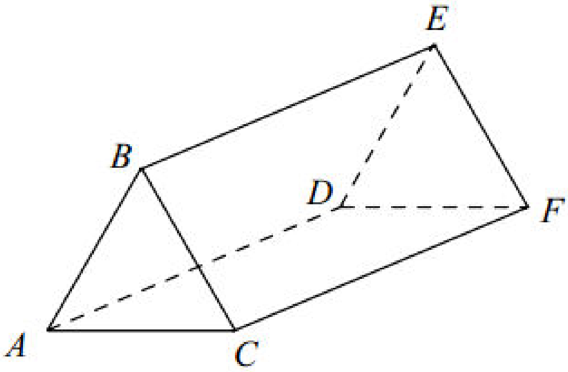
$$\left[\left(\frac{ab}{2}\right)^2 + \left(\frac{bc}{2}\right)^2 + \left(\frac{ac}{2}\right)^2\right]^{\frac{1}{2}}.$$

(4 marks)

Final Challenge

The triangular prism $ABCDEF$ has parallel triangular ends ABC and DEF, and the edges AD, BE and CF are parallel.

The coordinates of A, B, C and D are $(1, 2, 0)$, $(-1, 2, p)$, $(3, 0, 2)$ and $(4, 1, 5)$ respectively.



(a) Find the coordinates of the point E in terms of p. *(2 marks)*

(b) (i) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. *(3 marks)*

(ii) Hence find $\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})$, giving your answer in terms of p. *(2 marks)*

(c) (i) Determine the volume of the prism $ABCDEF$ when $p = 1$. *(2 marks)*

(ii) Describe the configuration of $ABCDEF$ when $p = -4$. *(2 marks)*