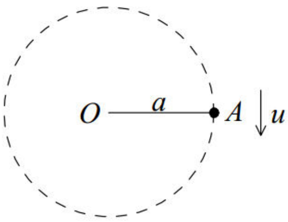
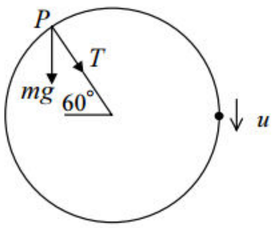
1 One end of a light inextensible string of length a is attached to a fixed point, O, and a particle of mass m is attached to the other end, A. The particle is held so that the string is taut and OA is horizontal. It is then projected vertically downwards with speed u as shown in the diagram.



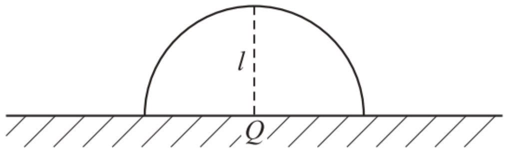
The string becomes slack when OA is inclined at an angle of 60° above the horizontal.

(a) Show that the speed of the particle when the string becomes slack is $\sqrt{\frac{\sqrt{3}}{2}ag}$. *(3 marks)*

(b) Hence find u in terms of a and g. *(4 marks)*

Question Number and part	Solution	Marks	Total	Comments
1(a)				
	At P when string is inclined at 60° $T = 0$ Resolving along string; $\frac{mv^2}{a} = mg\cos 30$	M1 A1		
	$v^2 = ag\frac{\sqrt{3}}{2} \quad \therefore\ v = \sqrt{\frac{\sqrt{3}}{2}ag}$	A1	3	
(b)	Using conservation of energy $\frac{1}{2}mu^2 - mga\sin 60 = \frac{1}{2}mv^2$	M1 A1		
	$v^2 = u^2 - 2ag\frac{\sqrt{3}}{2}$	M1		
	$ag\frac{\sqrt{3}}{2} = u^2 - ag\sqrt{3}$ $\therefore u^2 = \frac{3\sqrt{3}}{2}ag$ or $u = \sqrt{\frac{3\sqrt{3}}{2}ag}$	A1	4	
	Total		**7**	

3 A smooth hemisphere of radius l and centre Q lies with its plane face fixed to a horizontal surface. A particle, P, of mass m can move freely on the surface of the hemisphere.

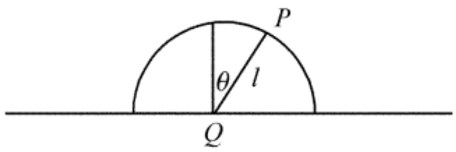
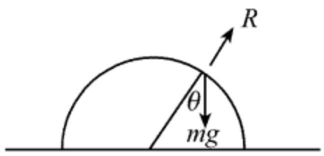


The particle is set in motion along the surface of the hemisphere with a speed, u, at the highest point of the hemisphere.

(a) Show that, while the particle is in contact with the hemisphere, the velocity of the particle when PQ makes an angle θ to the vertical, is

$$(u^2 + 2gl[1 - \cos\theta])^{\frac{1}{2}}.$$ *(4 marks)*

(b) Find, in terms of l, u and g, the cosine of the angle θ when the particle leaves the surface of the hemisphere. *(5 marks)*

Question Number and part	Solution	Marks	Total Marks	Comments
3(a)				
	By C of energy	M1		
	$\frac{1}{2}.mu^2 = \frac{1}{2}.mv^2 - mgl(1-\cos\theta)$	A1		
	$v^2 = u^2 + 2gl(1-\cos\theta)$	M1		
	$v = \{u^2 + 2gl(1-\cos\theta)\}^{1/2}$	A1	4	
(b)				
	Particle leaves surface when $R = 0$;	B1		
	$m \cdot \frac{v^2}{l} = mg\cos\theta - R$	M1 A1		
	i.e. $u^2 + 2gl(1-\cos\theta) = gl\cos\theta$	M1		
	$u^2 + 2gl = 3gl\cos\theta$			
	$\cos\theta = \frac{u^2 + 2gl}{3gl}$	A1	5	
	Total		**9**	

4 A bead, of mass m, is threaded onto a smooth circular ring, of radius r, which is fixed in a vertical plane. The bead is moving on the wire so that its speed at the lowest point of its path is four times its speed, v, at the highest point.

(a) Find v in terms of r and g. *(3 marks)*

(b) Find the reaction of the wire on the bead when the bead is at its lowest point. *(3 marks)*

Question Number and part	Solution	Marks	Total Marks	Comments
4(a)	Using conservation of energy between highest and lowest points			
	$\frac{1}{2}m(4v)^2 - \frac{1}{2}m(v)^2 = mg.2r$	M1		
	$15v^2 = 4gr$	A1		
	$v = \sqrt{\frac{4gr}{15}}$	A1	3	
(b)	Let R be the reaction of the wire on bead			
	Resolving vertically $R = mg + \frac{m(4v)^2}{r}$	M1 A1		
	$= mg + \frac{16m}{r}.\frac{4gr}{15}$			
	$= \frac{79}{15}mg$	A1	3	
	Total		**6**	

6 

The diagram shows a vertical cross section of a new adventure slide at a theme park. It consists of three sections AB, BC and CD.
Section AB is smooth and vertical and has length r.
Section BC is smooth and forms a quarter of a circle. This circle has centre O and radius r. The radius OB is horizontal and OC is vertical.
Section CD is rough, straight and horizontal. It is of length $4r$.

Steve, who has mass m, starts from rest at A and reaches speed u at the point B. He remains in contact with the surface until he reaches D.

It can be assumed that Steve can be modelled as a particle throughout the motion.

(a) Find u^2 in terms of g and r. *(2 marks)*

(b) Steve reaches the point P between B and C where angle $POB = \theta$, as shown in the diagram. His speed at P is v.

(i) Show that $v^2 = 2gr(1 + \sin\theta)$. *(4 marks)*

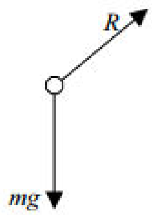
(ii) Draw a diagram showing the forces acting on Steve when he is at the point P. *(1 mark)*

(iii) Find an expression for the normal reaction, R, on Steve when he is at the point P. Give your answer in terms of m, g and θ. *(4 marks)*

(c) Show that, as Steve crosses C, there is a reduction in the normal reaction of magnitude $4mg$. *(2 marks)*

(d) Between C and D, Steve decelerates uniformly and comes to rest at the point D.

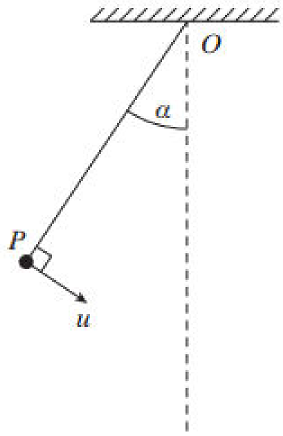
Find his retardation. *(3 marks)*

Q	Solution	Marks	Total	Comments
6 (a)	PE at A = KE at B			
	$\frac{1}{2}mu^2 = mgr$	M1		Attempt to use conservation of energy
	$\Rightarrow u^2 = 2gr$	A1	2	
	Alternative			
	Use $v^2 - u^2 = 2as$	*(M1)*		
	$v^2 - 0^2 = 2gr$			
	$u^2 = 2gr$	*(A1)*		
(b) (i)	Conservation of energy			
	Energy at B = Energy at P			
	$\frac{1}{2}mu^2 + mgr\sin\theta = \frac{1}{2}mv^2$	M1 A1 A1		Attempt to use conservation of energy A1 – one term correct A1 – all correct
	$v^2 = 2gr(1+\sin\theta)$	A1	4	Use of $u^2 = 2gr$ Printed answer
	Alternative			
	Energy at A = Energy at P			
	$mgr + mgr\sin\theta = \frac{1}{2}mv^2$	*(M1)* *(A1)* *(A1)*		Attempt to use conservation of energy A1 – one term correct A1 – all correct
	$v^2 = 2gr(1+\sin\theta)$	*(A1)*		Printed answer
(ii)		B1	1	Use mg or W
cont				

Q	Solution	Marks	Total	Comments
(iii)	Res force to centre			
	$= R - mg\sin\theta$	B1		Seen in equation or stated
	$F = ma$ radially gives,			
	$R - mg\sin\theta = \dfrac{mv^2}{r}$	M1A1$\sqrt{}$		Follow through error above
	$R = 2mg(1+\sin\theta) + mg\sin\theta$ or $mg(2+3\sin\theta)$	A1	4	Use of v^2 to find R, need not be simplified
(c)	$\theta = 90^\circ$, $R = 5mg$	M1		Either their R with $\theta = 90°$ or their $R - mg$ without $\theta = 90°$
	Reduces to mg after C since no longer circular motion.			
	$\therefore 4mg$ reduction	A1	2	Fully explained for A1
(d)	$u^2 = 4gr$	B1		Use of (b)(i) to find u^2
	Use $v^2 - u^2 = 2as$			
	$0^2 - 4gr = 2a(4r)$	M1		Suitable equation
	acc $= \dfrac{-g}{2}$			
	retardation $= \dfrac{g}{2}$	A1	3	Condone $+/-$
	Total		**16**	

Bonus question

7



The diagram shows a particle P, of mass m, which is attached by a light inextensible string, of length l, to a fixed support O. The particle moves in a vertical plane. It is initially set in motion with speed u at right angles to OP, from the position where OP makes an angle α with the downward vertical through O.

(a) Show that when OP makes an angle θ with the downward vertical through O the speed, v, of the particle is given by

$$v^2 = u^2 + 2gl(\cos\theta - \cos\alpha).$$ *(5 marks)*

(b) [In part (b) of this question the value of g should be taken to be $9.8\,\mathrm{m\,s^{-2}}$.]

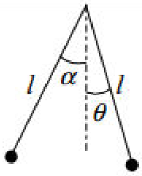
At an adventure playground a girl of mass 40 kg swings on the end of a rope of length 5 metres. The motion is in a vertical plane. Initially the rope makes an angle of 30° with the downward vertical and the girl has a speed of $2\,\mathrm{m\,s^{-1}}$ at right angles to the rope.

(i) Show that the maximum speed of the girl during the motion is approximately $4.1\,\mathrm{m\,s^{-1}}$. *(2 marks)*

(ii) Determine the maximum angle that the rope makes with the downward vertical. *(3 marks)*

(iii) Find the maximum tension in the rope. *(5 marks)*

(iv) State one modelling assumption which you have used in this problem. *(1 mark)*

Q	Solution	Marks	Total	Comments
7 (a)				
	Difference in height $h = l(\cos\theta - \cos\alpha)$	B1		stated or implied
	Change in KE $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	B1		use of $\frac{1}{2}mv^2$ seen
	Change in PE $= mgl(\cos\theta - \cos\alpha)$	B1√		use of mgh seen: ft error on h
	Using conservation of energy:			
	$mgl(\cos\theta - \cos\alpha) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	M1		forms equation using energy conservation
	$\therefore v^2 = u^2 + 2gl(\cos\theta - \cos\alpha)$	A1	5	AG
(b)(i)	$\theta = 0^\circ, u = 2, g = 9.8, l = 5, \alpha = 30^\circ$			
	$v_{\max} = \sqrt{2^2 + [2\times9.8\times5(\cos0 - \cos30)]}$	M1		use of correct values
	$= 4.1(\ldots)\ \text{m s}^{-1}$	A1	2	
(ii)	$v = 0, u = 2, g = 9.8, l = 5, \alpha = 30^\circ$	M1		$v = 0$ used
	$0 = 2^2 + [2^2 + 2\times9.8\times5(\cos\theta - \cos30)]$	A1		sub. other values and attempts rearrangement
	$\theta = 34^\circ$	A1	3	
(iii)	Consider the force along the radius $T - mg\cos\theta$ seen	B1		with/without $\theta = 0^\circ$
	Max T at lowest point so using $F = ma$ radially			
	$T - mg = \frac{mv^2}{l}$	M1 A1		forms equation; uses $\frac{mv^2}{r}$ and $\theta = 0^\circ$
	$m = 40, g = 9.8, v = 4.1, l = 5$	M1		uses correct values
	$\therefore T = (40\times9.8) + \frac{40\times4.1^2}{5}$			
	$= 530\text{ N}$ (2 s.f)	A1	5	AWRT
(iv)	Model as particle, inelastic string, no air resistance	B1	1	any sensible assumption
	Total		**16**	