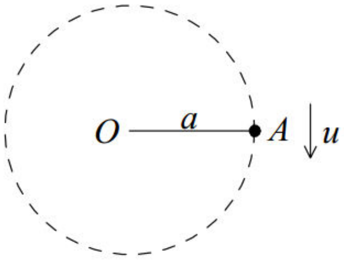
M2 Vertical Circular Motion Challenge

Challenge 1

One end of a light inextensible string of length a is attached to a fixed point, O, and a particle of mass m is attached to the other end, A. The particle is held so that the string is taut and OA is horizontal. It is then projected vertically downwards with speed u as shown in the diagram.



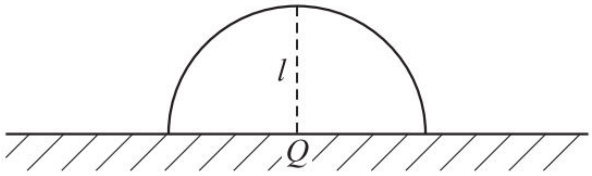
The string becomes slack when OA is inclined at an angle of $60°$ above the horizontal.

(a) Show that the speed of the particle when the string becomes slack is $\sqrt{\frac{\sqrt{3}}{2}ag}$. *(3 marks)*

(b) Hence find u in terms of a and g. *(4 marks)*

Challenge 2

A smooth hemisphere of radius l and centre Q lies with its plane face fixed to a horizontal surface. A particle, P, of mass m can move freely on the surface of the hemisphere.



The particle is set in motion along the surface of the hemisphere with a speed, u, at the highest point of the hemisphere.

(a) Show that, while the particle is in contact with the hemisphere, the velocity of the particle when PQ makes an angle θ to the vertical, is

$$(u^2 + 2gl[1 - \cos\theta])^{\frac{1}{2}}.$$

(4 marks)

(b) Find, in terms of l, u and g, the cosine of the angle θ when the particle leaves the surface of the hemisphere. *(5 marks)*

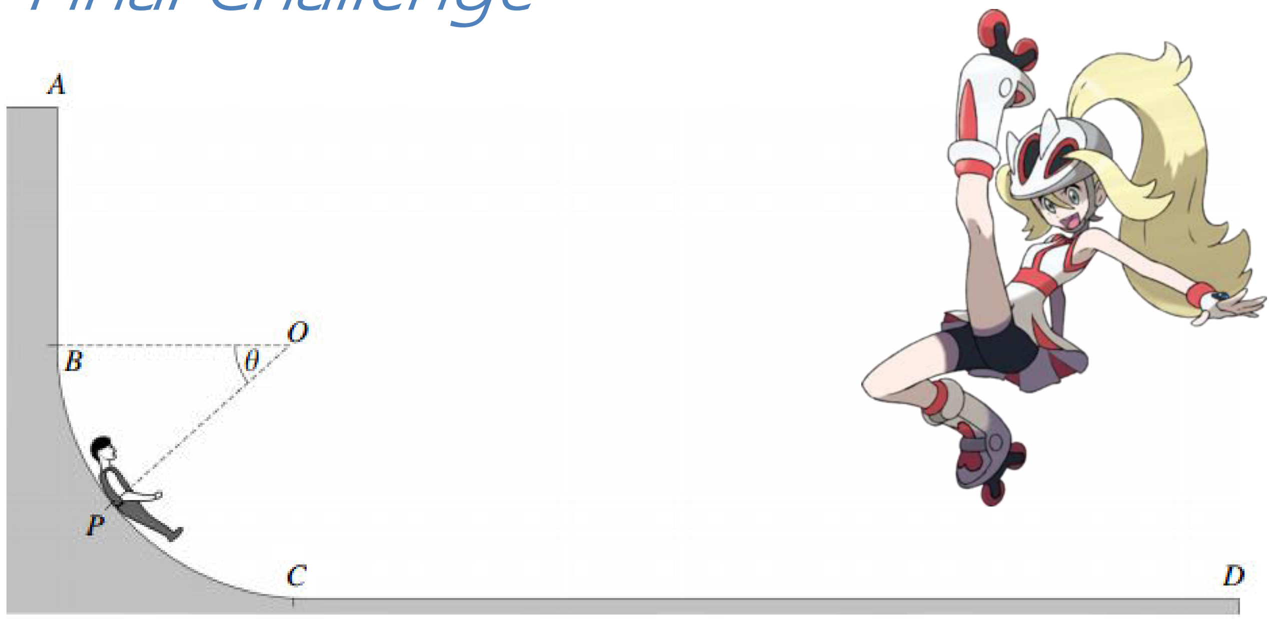
Challenge 3

A bead, of mass m, is threaded onto a smooth circular ring, of radius r, which is fixed in a vertical plane. The bead is moving on the wire so that its speed at the lowest point of its path is four times its speed, v, at the highest point.

(a) Find v in terms of r and g. *(3 marks)*

(b) Find the reaction of the wire on the bead when the bead is at its lowest point. *(3 marks)*

Final Challenge



The diagram shows a vertical cross section of a new adventure slide at a theme park. It consists of three sections AB, BC and CD.
Section AB is smooth and vertical and has length r.
Section BC is smooth and forms a quarter of a circle. This circle has centre O and radius r. The radius OB is horizontal and OC is vertical.
Section CD is rough, straight and horizontal. It is of length $4r$.

Steve, who has mass m, starts from rest at A and reaches speed u at the point B. He remains in contact with the surface until he reaches D.

It can be assumed that Steve can be modelled as a particle throughout the motion.

(a) Find u^2 in terms of g and r. *(2 marks)*

(b) Steve reaches the point P between B and C where angle $POB = \theta$, as shown in the diagram. His speed at P is v.

(i) Show that $v^2 = 2gr(1 + \sin\theta)$. *(4 marks)*

(ii) Draw a diagram showing the forces acting on Steve when he is at the point P. *(1 mark)*

(iii) Find an expression for the normal reaction, R, on Steve when he is at the point P. Give your answer in terms of m, g and θ. *(4 marks)*

(c) Show that, as Steve crosses C, there is a reduction in the normal reaction of magnitude $4mg$. *(2 marks)*

(d) Between C and D, Steve decelerates uniformly and comes to rest at the point D.

Find his retardation. *(3 marks)*