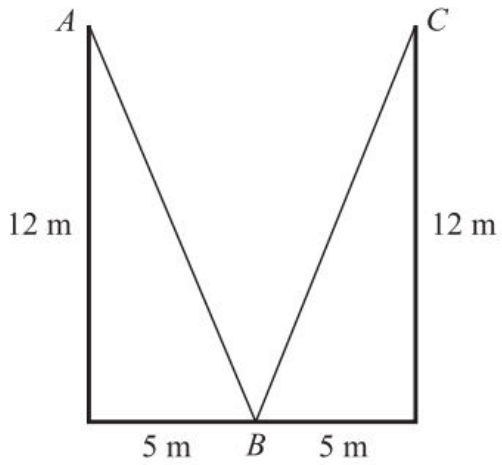
M2 Work and Energy Challenge

Challenge 1

A "reverse bungee jump" consists of a 12 metre length of elastic rope, that is stretched into a "V" shape ABC on a frame, as shown in the diagram. The ends of the elastic rope are fixed to the frame at the points A and C.



A student, of mass 85 kg, is attached to the midpoint of the elastic rope at B. The modulus of elasticity of the elastic rope is 1500 N.

(a) Show that the elastic potential energy of the elastic rope in the initial position shown in the diagram is 12250 J. *(3 marks)*

The middle of the rope is then released from B and the student moves vertically upwards.

(b) Find the speed of the student, when at a height of 12 metres above B. *(3 marks)*

The student reaches his maximum height before the rope becomes taut again.

(c) Find the maximum height of the student above B during the motion. *(2 marks)*

Challenge 2

A ball is projected vertically upwards, from ground level, with an initial speed of $18\,\text{m}\,\text{s}^{-1}$. The ball has a mass of 0.3 kg. Assume that the force of gravity is the only force acting on the ball after it is projected.

(a) Calculate the initial kinetic energy of the ball. *(2 marks)*

(b) By using conservation of energy, find the maximum height of the ball above ground level. *(2 marks)*

(c) Find the kinetic energy and the speed of the ball when it is at a height of 2 metres above ground level. *(5 marks)*

Challenge 3

An elastic string has modulus of elasticity 12 N and natural length 0.5 metres. A particle of mass 0.5 kg is attached to one end of the string. The other end of the string is attached to a fixed point P. The particle is pulled down until it is 1.5 metres below P.

(a) Calculate the elastic potential energy of the string when the particle is 1.5 metres below P.
(2 marks)

(b) The particle is released.

(i) Show that the kinetic energy of the particle is 7.1 J, when the string becomes slack.
(2 marks)

(ii) Find the kinetic energy of the particle when it is 0.5 metres above P. *(2 marks)*

(iii) Find the maximum height of the particle above P. *(7 marks)*

Final Challenge

An elastic string has natural length 2 metres and modulus of elasticity λ newtons. One end of the string is fixed at the point O, and a particle of mass 20 kg is attached to the other end of the string.

(a) When in equilibrium the particle is 2.7 metres below O. Show that $\lambda = 560$. *(3 marks)*

(b) The particle is now held at O and released from rest. The maximum length of the string in the subsequent motion is L.

(i) Show that L satisfies the equation

$$5L^2 - 27L + 20 = 0$$ *(5 marks)*

(ii) Find the maximum length of the string. *(3 marks)*